

# Deconfinement Transition of AdS/QCD at $\mathcal{O}(\alpha'^3)$

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## Abstract

We study the confinement/deconfinement phase transition of holographic AdS/QCD models by using Ricci flat  $AdS_5$  black holes up to  $\mathcal{O}(\alpha'^3)$ , which corresponds to the  $\lambda$  expansion correction in the dual field theory to  $\lambda^{-3/2}$ , where  $\lambda$  is the 't Hooft coupling constant. We consider two cases: one is the hard-wall AdS/QCD model where a small radius region of the  $AdS_5$  is removed; the other is the case where one of spatial coordinates for the  $AdS_5$  space is compactified, resulting in Witten's QCD model in  $2 + 1$  dimensions. We find that in the hard-wall AdS/QCD model, the deconfinement temperature decreases when the  $\lambda$  expansion corrections are taken into account, while in Witten's QCD model, the deconfinement transition always happens when the ratio of inverse temperature  $\beta$  to the period  $\beta_s$  of the compactified coordinate decreases to one,  $\beta/\beta_s = 1$ , the same as the case without the  $\mathcal{O}(\alpha'^3)$  correction.

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# 1 Introduction

The remarkable AdS/CFT correspondence [1] conjectures that string/M theory in an anti-de Sitter space (AdS) times a compact manifold is dual to a large  $N$  strongly coupling conformal field theory (CFT) residing on the boundary of the AdS space. At finite temperature, Witten [2] argued that the thermodynamics of black holes in AdS space can be identified with that of the dual strongly coupling field theory in the high temperature limit. In addition, it is well known that there exists a phase transition between the Schwarzschild-AdS black hole and thermal AdS space, the so-called Hawking-Page phase transition [3]: the black hole phase dominates the partition function in the high temperature limit, while the thermal AdS space dominates in the low temperature limit. This phase transition is a first order one, and is interpreted as the confinement/deconfinement phase transition in the dual field theory [2].

A special example of the AdS/CFT correspondence is that type IIB string theory in  $AdS_5 \times S^5$  is dual to a four dimensional  $\mathcal{N} = 4$  supersymmetric Yang-Mills (SYM) theory on the boundary of  $AdS_5$ . Acting as the near horizon geometry of D3-branes, the  $AdS_5$  space is the one in the Poincare coordinates and dual field theory resides on a manifold with topology  $R \times R^3$ . In that case, the dual field theory is always in the deconfinement phase [4, 5]. In order to realize a confined phase, the authors of [6] proposed a simple model, where a small radius region is removed from the  $AdS_5$  space in the Poincare coordinates without compact directions. Note that the radial coordinate in the AdS space is dual to an energy scale in the dual field theory. Therefore removing a small radius region of AdS space corresponds to introducing an IR cutoff and a mass gap in the dual theory. The model in [6] is called hard-wall AdS/QCD model in  $3 + 1$  dimensions. Although the hard wall model is somewhat rough, it turns out that the model can give some realistic, semiquantitative description of low energy QCD [7, 8].

Recently, Herzog [9] has shown that in this simple hard wall model of AdS/QCD, the confinement/deconfinement phase transition occurs via a first order Hawking-Page phase transition between the low temperature thermal AdS space and high temperature AdS black hole in the Poincare coordinates. This deconfinement phase transition has been studied in various cases [10]-[17]. For instance, in [16] the deconfinement transition has been studied for the AdS/QCD model in curved spaces and with chemical potential; the authors of [17] have discussed the effect of matter on the deconfinement temperature.

Another scenario to realize the confinement of gauge field was proposed by Witten [2]: Consider one of spatial directions on the world volume of D3-branes is compactified, and fermions along that direction are anti-periodic so that supersymmetry is broken. In that

case, the near horizon geometry  $AdS_5 \times S^5$  can be regarded as gravity dual of low-energy QCD model in  $2+1$  dimensions. When one of spatial directions is compactified, Horowitz and Myers [18] have shown that there is a lower mass solution than the pure AdS space. This solution is called AdS soliton. Viewed AdS soliton as a reference background, it was shown that there is a Hawking-Page phase transition between Ricci flat AdS black holes and AdS soliton [19]-[24]. It is found that the deconfinement transition is determined by the ratio of inverse Hawking temperature of the Ricci flat black hole to the period of the compactified direction.

In this note we will discuss the deconfinement transition of the above two holographic AdS/QCD models. Specially we pay attention on the effect of the terms  $\mathcal{O}(\alpha'^3)$  on the deconfinement temperature. In type IIB supergravity, the leading correction of  $\alpha'$  expansion to the low-energy effective action is of the form  $\alpha'^3 R^4$ . The  $\alpha'$  expansion is dual to  $\lambda$  expansion in the dual field theory, here  $\lambda$  is the 't Hooft coupling constant of gauge field. Thus the term  $\mathcal{O}(\alpha'^3)$  in the supergravity action corresponds to the correction  $\lambda^{-3/2}$  in the QCD models.

The organization of this paper is as follows. In the next section we first review the  $AdS_5$  black hole solution, up to the correction  $\mathcal{O}(\alpha'^3)$ . In Sec. III we discuss the deconfinement phase transition of the hard-wall AdS/QCD model. Sec. IV is devoted to the case of Witten's QCD model in  $2+1$  dimensions. We end the paper with conclusion and discussion in Sec. V.

## 2 $AdS_5$ black holes at $\mathcal{O}(\alpha'^3)$

The near horizon geometry of black D3-branes can be described by a Ricci flat  $AdS_5$  black hole times a constant radius  $S^5$

$$ds^2 = \frac{r^2}{L^2}(-f dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + \frac{L^2}{r^2} f^{-1} dr^2 + L^2 d\Omega_5^2, \quad (2.1)$$

where  $f = 1 - r_0^4/r^4$ ,  $L^4 = g_{YM}^2 N \alpha'^2$ ,  $N$  is the number of D3-branes and is the rank of dual gauge field with SYM coupling constant  $g_{YM}^2 = 4\pi g_s$ ,  $\lambda = g_{YM}^2 N$  is the 't Hooft coupling constant for the  $SU(N)$  gauge theory<sup>1</sup>,  $r_0$  is the mass parameter of the solution.

The black hole horizon is located at  $r = r_0$ . The associated inverse Hawking temperature of the black hole solution (2.1) is

$$\beta_0 = \pi L^2 / r_0. \quad (2.2)$$

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<sup>1</sup>Here the notations of the SYM coupling  $g_{YM}^2$  and the 't Hooft coupling constant are different from those in [25] by a factor 2, for more details see [33].

The 5-dimensional effective action obtained from 10-dimensional type IIB supergravity compactifying on  $S^5$  is

$$I_0 = -\frac{1}{16\pi G_5} \int d^5x \sqrt{g_5} \left( R_5 + \frac{12}{L^2} \right). \quad (2.3)$$

Calculating the action yields the free energy and entropy of the black hole [25]

$$F_0 = -\frac{\pi^2}{8} N^2 V_3 T_0^4, \quad S_0 = \frac{\pi^2}{2} N^2 V_3 T_0^3 \quad (2.4)$$

where  $V_3$  is the volume of space spanned by  $x_1, x_2$  and  $x_3$ . On the other hand, the entropy of the weak coupling limit of the  $SU(N)$  SYM theory, where it reduces to that of  $8N^2$  free massless bosons and fermions, is

$$S_{\text{YM}} = \frac{2\pi^2}{3} N^2 V_3 T_0^3. \quad (2.5)$$

This means that  $S_0 = \frac{3}{4} S_{\text{YM}}$ . Note that according to the dictionary of AdS/CFT correspondence, the black hole entropy is equivalent to that of SYM theory in the limit of  $N \rightarrow \infty$  and large 't Hooft coupling constant  $\lambda$ . The relations  $L^4/l_p^4 \sim N$  and  $L^4/l_s^4 \sim g_{\text{YM}}^2 N$  tell us that only in that limit both the  $\alpha'$  and loop corrections to the solution (2.1) can be neglected.

In [25], the  $\alpha'$  corrections have been considered in the dual gravity side. In type IIB supergravity, the leading correction terms are of the form  $\alpha'^3 R^4$ . The tree level type IIB string effective action has the following form

$$I = -\frac{1}{16\pi G_{10}} \int d^{10}x \sqrt{g} \left( R - \frac{1}{2} (\partial\phi)^2 - \frac{1}{4 \cdot 5!} (F_5)^2 + \dots + \gamma e^{-3\phi/2} W + \dots \right), \quad (2.6)$$

where  $\gamma = \frac{1}{8} \zeta(3) (\alpha')^3$ ,  $F_5$  is assumed self-dual, dots stand for other terms depending on antisymmetric tensor field strengths and derivatives of dilaton, and

$$\begin{aligned} W = & R^{hmnk} R_{pmnq} R_h^{rsp} R_{rsk}^q + \frac{1}{2} R^{hkmn} R_{pqmn} R_h^{rsp} R_{rsk}^q \\ & + \text{terms depending on the Ricci tensor.} \end{aligned}$$

The field redefinition ambiguity allows one to change the coefficients of terms involving the Ricci tensor (in essence, ignoring other fields, one may use  $R_{mn} = 0$  to simplify the structure of  $W$  as the graviton legs in the 4-point string amplitude are on-shell) [25]. Thus one can find a scheme where  $W$  in (2.6) depends only on the Weyl tensor

$$W = C^{hmnk} C_{pmnq} C_h^{rsp} C_{rsk}^q + \frac{1}{2} C^{hkmn} C_{pqmn} C_h^{rsp} C_{rsk}^q. \quad (2.7)$$

The form of  $W$  is special in the sense that the  $AdS_5 \times S^5$  is still the solution of the action (2.6) with self-dual  $F_5$  and a constant dilaton. The corrected free energy turns out to be of the form

$$F = F_0 + \delta F = -\frac{\pi^2}{8} N^2 V_3 T_0^4 \left( 1 + \frac{15}{8} \zeta(3) \lambda^{-3/2} \right), \quad (2.8)$$

and the corrected entropy of the black hole

$$S = \frac{\pi^2}{2} N^2 V_3 T_0^3 \left( 1 + \frac{15}{8} \zeta(3) \lambda^{-3/2} \right). \quad (2.9)$$

Indeed one can see that the leading correction is positive. If we write the free energy in the following form

$$F = -f(\lambda) \frac{\pi^2}{6} N^2 V_3 T_0^4, \quad (2.10)$$

it is expected that the function  $f(\lambda)$  approaches 1 for small coupling  $\lambda$ , then for large coupling, one has [25]

$$f(\lambda) = \frac{3}{4} + \frac{45}{32} \zeta(3) \lambda^{-3/2} + \dots \quad (2.11)$$

The  $R^4$  correction to the AdS black holes metric and their thermodynamics have also been discussed in various cases [26]-[30]. The corrections from the  $R^4$  terms in the non-commutative SYM theory have been discussed in [31].

The correction to the  $AdS_5$  black hole metric due to the  $R^4$  term in the action (2.6) can be obtained by following [25]. We consider the following metric ansatz for the 10-dimensional metric in the Einstein frame,

$$ds_{10}^2 = e^{-\frac{10}{3}\nu(x)} g_{5mn} dx^m dx^n + e^{2\nu(x)} d\Omega_5^2, \quad (2.12)$$

where we have set  $L = 1$ . Thus in what follows,  $\gamma = \frac{1}{8} \zeta(3) \lambda^{-3/2}$ . Considering the standard ansatz for the field strength and compactifying on  $S^5$ , one can obtain the 5-dimensional effective action

$$I_5 = -\frac{1}{16\pi G_5} \int d^5x \sqrt{g_5} \left[ R_5 - \frac{1}{2} (\partial\phi)^2 - \frac{40}{3} (\partial\nu)^2 - V(\nu) + \gamma e^{10\nu - \frac{3}{2}\phi} \left( W + \mathcal{O}((\partial\nu)^2) \right) \right], \quad (2.13)$$

where

$$V(\nu) = 8e^{-\frac{40}{3}\nu} - 20e^{-\frac{16}{3}\nu}.$$

Up to the order  $\mathcal{O}(\gamma)$ , one has

$$\nu(r) = \frac{15\gamma}{32} \frac{r_0^8}{r^8} \left( 1 + \frac{r_0^4}{r^4} \right) + \mathcal{O}(\gamma^2), \quad (2.14)$$

while the dilaton field  $\phi_1 = \phi - \phi_0$  is

$$\phi_1 = -\frac{45}{8}\gamma \left( \frac{r_0^4}{r^4} + \frac{r_0^8}{2r^8} + \frac{r_0^{12}}{3r^{12}} \right) + \mathcal{O}(\gamma^2). \quad (2.15)$$

The 5-dimensional metric is given by [25]

$$ds_5^2 = H^2(K^2 d\tau^2 + P^2 dr^2 + dx_1^2 + dx_2^2 + dx_3^2), \quad (2.16)$$

with

$$H = r, \quad K = e^{a+4b}, \quad P = e^b, \quad (2.17)$$

and

$$\begin{aligned} a &= -2 \ln r + \frac{5}{2} \ln(r^4 - r_0^4) - \frac{15}{2}\gamma \left( 25 \frac{r_0^4}{r^4} + 25 \frac{r_0^8}{r^8} - 79 \frac{r_0^{12}}{r^{12}} \right) + \mathcal{O}(\gamma^2), \\ b &= -\frac{1}{2} \ln(r^4 - r_0^4) + \frac{15}{2}\gamma \left( 5 \frac{r_0^4}{r^4} + 5 \frac{r_0^8}{r^8} - 19 \frac{r_0^{12}}{r^{12}} \right) + \mathcal{O}(\gamma^2). \end{aligned} \quad (2.18)$$

Note that the leading order for both of the dilaton field  $\phi$  and scalar  $\nu$  is  $\mathcal{O}(\gamma)$ . Therefore the action (2.13) can be reduced to

$$I_5 = -\frac{1}{16\pi G_5} \int d^5x \sqrt{g_5} (R_5 + 12 + \gamma W), \quad (2.19)$$

up to the leading order correction of the  $\gamma$  term. Namely in this order the dilaton field and scalar field  $\nu$  have no contribution to the Euclidean action and free energy. For the black hole solution (2.16), the Hawking temperature is

$$T \equiv 1/\beta = \frac{r_0}{\pi} (1 + 15\gamma) + \mathcal{O}(\gamma^2). \quad (2.20)$$

### 3 Deconfinement transition in the hard-wall AdS/QCD

Let us notice that even with the  $R^4$  correction, the free energy (2.8) is always negative, which means that the dual field theory is in the deconfinement phase. It is true even in the zero-temperature phase, which can be verified by calculating the quark-anti-quark potential in the dual gravity configuration [4, 5]. Recently it has been found that one can realize a confined phase of gauge theory in the dual  $AdS_5$  gravity configuration by simply removing a small radius region of  $AdS_5$  spacetime [6], which is equivalent to introducing an IR cutoff in the dual field theory. The  $AdS_5$  spacetime with an IR cutoff is called hard-wall AdS/QCD model. More recently it has been found that the deconfinement phase transition of the hard-wall AdS/QCD model can be realized via a first-order Hawking-Page

phase transition between a thermal AdS space and a Ricci flat Schwarzschild- $AdS_5$  black hole [9]. In the following, we will consider the effect of the  $R^4$  term on the deconfinement phase transition.

Now we calculate the Euclidean action associated with the black hole (2.16). Due to an infinite volume, the action (2.19) is divergent. To get a finite result, one has to regularize the action by subtracting the contribution of a suitable reference background. Here the suitable reference background is obviously the one (2.16) with  $r_0 = 0$ . Namely we take the  $AdS_5$  spacetime in the Poincare coordinates as the reference background. To finish this calculation, we introduce an UV boundary at  $r = R$  ( $R \gg r_0$ ) (Note that in the coordinates (2.16), the radial coordinate  $r$  is equivalent to an energy scale in the dual field theory). On the other hand, we introduce an IR cutoff at  $r = x_0$  in order to realize the confinement in the hard-wall AdS/QCD model. In the black hole phase, the IR cutoff is taken as  $r_{\max} = \max(r_0, x_0)$ . Therefore, one has

$$I_{5BH} = -\frac{N^2}{8\pi^2} V_3 \beta \int_{r_{\max}}^R dr \sqrt{g_5} (R_5 + 12 + \gamma W). \quad (3.1)$$

For the reference background, in order that the black hole solution can be embedded into the reference background, the Euclidean time for the background has to satisfy the following condition at the UV boundary

$$\beta' \sqrt{g_{00}(r_0 = 0, r = R)} = \beta \sqrt{g_{00}(r_0, r = R)}, \quad (3.2)$$

which gives

$$\beta' = \beta \left( 1 - \frac{1}{2}(1 + 75\gamma) \frac{r_0^4}{R^4} + \mathcal{O}\left(\frac{r_0^8}{R^8}\right) \right). \quad (3.3)$$

Thus the contribution from the background is

$$\begin{aligned} I_{5REF} &= -\frac{N^2}{8\pi^2} V_3 \beta' \int_{x_0}^R dr \sqrt{g_5} (R_5 + 12 + \gamma W) \\ &= \frac{N^2}{4\pi^2} V_3 \beta' (R^4 - x_0^4) \\ &= \frac{N^2}{4\pi^2} V_3 \beta \left( R^4 - \frac{1}{2}(1 + 75\gamma) r_0^4 - x_0^4 \right). \end{aligned} \quad (3.4)$$

Note that the  $R^4$  term ( $\gamma W$ ) gives no contribution to the action of the reference background. The Euclidean action for the black hole solution is

$$\begin{aligned} I_{5BH} &= -\frac{N^2}{8\pi^2} V_3 \beta \int_{r_{\max}}^R dr \left( -8r^3 + \gamma \left( \frac{360r_0^{16}}{r^{13}} + \frac{960r_0^{12}}{r^9} \right) + \mathcal{O}(\gamma^2) \right) \\ &= \frac{N^2}{4\pi^2} V_3 \beta \left( R^4 - r_{\max}^4 + 15r_0^{12} \gamma \left( r_0^4 \left( \frac{1}{R^{12}} - \frac{1}{r_{\max}^{12}} \right) + 4 \left( \frac{1}{R^8} - \frac{1}{r_{\max}^8} \right) \right) \right). \end{aligned} \quad (3.5)$$

Subtracting the contribution of the reference background from (3.5) and taking  $R \rightarrow \infty$ , one obtains the Euclidean action of the black hole

$$\mathcal{I} = \frac{N^2}{4\pi^2} V_3 \beta \left( -r_{\max}^4 + \frac{1}{2}(1 + 75\gamma)r_0^4 + x_0^4 - 15r_0^{12}\gamma \left( \frac{r_0^4}{r_{\max}^{12}} + \frac{4}{r_{\max}^8} \right) \right). \quad (3.6)$$

Now it is the position to discuss the action.

### 3.1 No IR cutoff

Let us first consider the case without the IR cutoff, namely  $x_0 = 0$  and  $r_{\max} = r_0$ . In this case, one has

$$\mathcal{I} = -\frac{N^2}{8\pi^2} V_3 \beta r_0^4 (1 + 75\gamma) = -\frac{N^2 \pi^2}{8} V_3 T^3 (1 + 15\gamma). \quad (3.7)$$

This is exactly the result obtained in [25], and this gives the free energy in (2.8). Therefore no Hawking-Page phase transition will happen in this case.

### 3.2 No higher order term

In the case without the  $R^4$  term, namely  $\gamma = 0$ , one has

$$\mathcal{I} = -\frac{N^2}{8\pi^2} V_3 \beta \left( 2r_{\max}^4 - r_0^4 - 2x_0^4 \right), \quad (3.8)$$

which reproduces the results in [9, 16]: when  $x_0 > r_0$ , one has  $r_{\max} = x_0$ ; the action is always positive, and no Hawking-Page transition will occur. In contrast, a Hawking-Page transition can happen when  $x_0 < r_0$ . In the latter case, one has  $r_{\max} = r_0$ ; the action is negative for  $r_0^4 > 2x_0^4$  and positive for  $r_0^4 < 2x_0^4$ ; the action has the form

$$\mathcal{I} = -\frac{N^2}{8\pi^2} V_3 \beta \left( r_0^4 - 2x_0^4 \right). \quad (3.9)$$

The deconfinement temperature is

$$T_c = 2^{\frac{1}{4}} \frac{x_0}{\pi}. \quad (3.10)$$

This is precisely the result in [9]. Note that here  $x_0 = 1/z_0$  in [9]. Using the lightest  $\rho$  meson mass, one may conclude  $z_0 = 1/(323 \text{ MeV})$ . Note the fact that the value  $z_0$  is obtained by using the Bessel function in  $AdS_5$  space [9], and the  $W$  term does not change the geometry of  $AdS_5$ . We conclude that the correction term  $W$  will not give rise to any change of the value  $z_0$ .



### 3.3 Effects of IR cutoff and higher order term

Now we consider the case where both the IR cutoff and the  $R^4$  term are present. Here we have two subcases:

1) If the IR cutoff  $x_0 > r_0$ , then one has  $r_{\max} = x_0$ , and the action reduces to

$$\begin{aligned}\mathcal{I} &= \frac{N^2}{8\pi^2} V_3 \beta r_0^4 \left( 1 + 75\gamma - 30\gamma \left( \frac{r_0^{12}}{x_0^{12}} + \frac{4r_0^8}{x_0^8} \right) \right) \\ &= \frac{N^2 \pi^2}{8} V_3 T^3 \left( 1 + 15\gamma - 30\gamma \left( \frac{T^{12}}{T_{IR}^{12}} + 4 \frac{T^8}{T_{IR}^8} \right) \right),\end{aligned}\quad (3.11)$$

where we have defined  $T_{IR} = x_0/\pi$ . Although  $\gamma < 1$  and  $T/T_{IR} < 1$ , at first sight it appears that the action (3.11) can change the sign; if so, it would imply that a Hawking-Page phase transition would take place. However, this conclusion is unreliable because in getting the action (3.11), we have treated the terms concerning  $\gamma$  as perturbative terms, which indicates that the action is valid only when the second and third terms are much less than the first term in (3.11). In that sense, the action (3.11) is always positive, and the thermal  $AdS_5$  space with an IR cutoff dominates in the lower temperature phase, the dual field theory is always in the confined phase. What we can conclude here is that the action gets negative contribution from the higher order effect.

2) If  $x_0 < r_0$ , one has  $r_{\max} = r_0$ . In that case, the action becomes

$$\begin{aligned}\mathcal{I} &= -\frac{N^2}{8\pi^2} V_3 \beta \left( (1 + 75\gamma) r_0^4 - 2x_0^4 \right) \\ &= -\frac{N^2 \pi^2}{8} V_3 T^3 \left( 1 + 15\gamma - \frac{2x_0^4}{\pi^4 T^4} \right).\end{aligned}\quad (3.12)$$

The action can change its sign. A Hawking-Page phase transition happens when  $\mathcal{I} = 0$ . In the high temperature phase, the action is negative and dual field theory is in the deconfinement phase, while it is in the confined phase in the low temperature case. Clearly the phase transition temperature is

$$T_c = \frac{2^{1/4} x_0}{\pi} \left( 1 - \frac{15}{4} \gamma \right). \quad (3.13)$$

When  $\gamma = 0$ , it reduces to the one (3.10). We see from (3.13) that the deconfinement temperature decreases if we incorporate the correction  $\mathcal{O}(\alpha'^3)$ .

The energy of the dual field theory is

$$E \equiv \frac{\partial \mathcal{I}}{\partial \beta} = \frac{3N^2 \pi^2}{8} V_3 T^4 \left( 1 + 15\gamma + \frac{2x_0^4}{3\pi^4 T^4} \right), \quad (3.14)$$

while its entropy is

$$S = \frac{N^2 \pi^2}{2} V_3 T^3 (1 + 15\gamma). \quad (3.15)$$

We see from (3.14) that indeed introducing an IR cutoff is equivalent to a mass gap; the energy does not vanish even when the temperature approaches zero, while the entropy is still the same as the case without the IR cutoff.

## 4 Deconfinement transition in Witten's QCD Model

In this section we discuss the deconfinement transition in Witten's QCD model in  $2 + 1$  dimensions. To this aim, we have to first get the corresponding soliton solution, which will act as the reference background. The deformed AdS soliton at the order  $\mathcal{O}(\alpha'^3)$  can be obtained by continuing the Euclidean black hole solution (2.16) in the way  $x_1 \rightarrow it$

$$ds_5^2 = H_s^2 (K_s^2 d\tau^2 + P_s^2 dr^2 - dt^2 + dx_2^2 + dx_3^2), \quad (4.1)$$

where  $H_s$ ,  $K_s$  and  $P_s$  are still given by  $H$ ,  $K$  and  $P$  in (2.17), but replacing  $r_0$  by a new constant  $r_s$  in (2.18). Now the coordinate  $\tau$  is a spatial one. In order to remove the conical singularity in the plane spanned by  $\tau$  and  $r$ , the coordinate  $\tau$  has to be a periodic one with period

$$\beta_s = (1 - 15\gamma) \frac{\pi}{r_s} + \mathcal{O}(\gamma^2). \quad (4.2)$$

Let us next consider the black hole solution in (2.16)

$$ds_5^2 = H^2 (-K^2 dt^2 + P^2 dr^2 + dx_1^2 + dx_2^2 + dx_3^2), \quad (4.3)$$

with a compactified coordinate  $x_1$  with period  $\eta$ . The dual field theory resides on a manifold with topology  $R^1 \times S^1 \times R^2$ , where  $R^1$  stands for time,  $S^1$  represents the coordinate  $x_1$  and  $R^2$  is for the coordinates  $x_2$  and  $x_3$ . In this case, a natural reference background is just the one (4.3) with  $r_0 = 0$ , namely the  $AdS_5$  space in the Poincare coordinates, but with a compactified coordinate  $x_1$ , like the situation discussed in the previous section. However, as shown in [18], the AdS soliton has a less energy than the  $AdS_5$  space. In our case, the corresponding AdS soliton solution is the one given by (4.1), up to  $\mathcal{O}(\alpha'^3)$ . Therefore we will study the Euclidean action of the black hole (4.3) by regarding the AdS soliton (4.1) as the reference background. In order that the black hole solution can be embedded into the reference background, at the UV boundary  $r = R$  ( $R \gg r_0$  and  $R \gg r_s$ ), we must have the matching conditions

$$\beta H K(r = R) = \beta_b H_s(r = R), \quad \eta H(r = R) = \beta_s H_s K_s(r = R), \quad (4.4)$$

where  $\beta$  is the inverse temperature of the black hole (4.3), which is still given by (2.20), while  $\beta_b$  is the period of the Euclidean time for the soliton solution (4.1). Thus we have

$$\begin{aligned}\beta_b &= \beta \left( 1 - \frac{1}{2}(1 + 75\gamma) \frac{r_0^4}{R^4} + \mathcal{O}\left(\frac{r_0^8}{R^8}\right) \right), \\ \eta &= \beta_s \left( 1 - \frac{1}{2}(1 + 75\gamma) \frac{r_s^4}{R^4} + \mathcal{O}\left(\frac{r_s^8}{R^8}\right) \right).\end{aligned}\quad (4.5)$$

The Euclidean action for the black hole is

$$\begin{aligned}I_{5BH} &= -\frac{N^2}{8\pi^2} V_2 \eta \beta \int_{r_0}^R dr \left( -8r^3 + \gamma \left( \frac{360r_0^{16}}{r^{13}} + \frac{960r_0^{12}}{r^9} \right) + \mathcal{O}(\gamma^2) \right) \\ &= \frac{N^2}{4\pi^2} V_2 \beta \eta \left( R^4 - r_0^4 + 15r_0^{12} \gamma \left( r_0^4 \left( \frac{1}{R^{12}} - \frac{1}{r_0^{12}} \right) + 4 \left( \frac{1}{R^8} - \frac{1}{r_0^8} \right) \right) \right) \\ &= \frac{N^2}{4\pi^2} V_2 \beta \beta_s \left( R^4 - (1 + 75\gamma) r_0^4 - \frac{1}{2}(1 + 75\gamma) r_s^4 \right),\end{aligned}\quad (4.6)$$

where  $V_2$  is the volume for the space spanned by coordinates  $x_2$  and  $x_3$  and in the last equality we have dropped terms disappearing in the large  $R$  limit. On the other hand, the Euclidean action for the soliton is

$$\begin{aligned}I_{5REF} &= -\frac{N^2}{8\pi^2} V_2 \beta_b \beta_s \int_{r_s}^R dr \left( -8r^3 + \gamma \left( \frac{360r_0^{16}}{r^{13}} + \frac{960r_0^{12}}{r^9} \right) + \mathcal{O}(\gamma^2) \right) \\ &= \frac{N^2}{4\pi^2} V_2 \beta_b \beta_s \left( R^4 - r_s^4 + 15r_s^{12} \gamma \left( r_s^4 \left( \frac{1}{R^{12}} - \frac{1}{r_s^{12}} \right) + 4 \left( \frac{1}{R^8} - \frac{1}{r_s^8} \right) \right) \right) \\ &= \frac{N^2}{4\pi^2} V_2 \beta \beta_s \left( R^4 - (1 + 75\gamma) r_s^4 - \frac{1}{2}(1 + 75\gamma) r_0^4 \right).\end{aligned}\quad (4.7)$$

In the last equality we have considered the large  $R$  limit, once again. The difference between the Euclidean actions for the black hole and soliton is

$$\begin{aligned}\mathcal{I} &= -\frac{N^2}{8\pi^2} V_2 \beta \beta_s (1 + 75\gamma) (r_0^4 - r_s^4) \\ &= -\frac{N^2 \pi^2}{8} V_2 \beta^{-3} \beta_s (1 + 15\gamma) \left( 1 - \frac{\beta^4}{\beta_s^4} \right).\end{aligned}\quad (4.8)$$

The energy of the black hole is, via  $E = \partial \mathcal{I} / \partial \beta$ ,

$$E = \frac{N^2 \pi^2}{8} V_2 \beta^{-4} \beta_s (1 + 15\gamma) \left( 3 + \frac{\beta^4}{\beta_s^4} \right),\quad (4.9)$$

and the entropy associated with the black hole is

$$S = \frac{N^2 \pi^2}{2} V_2 \beta^{-3} \beta_s (1 + 15\gamma).\quad (4.10)$$

We see from the action (4.8) that the action changes its sign at  $\beta = \beta_s$ : when  $\beta < \beta_s$ , it is negative while positive as  $\beta > \beta_s$ . This implies that a Hawking-Page phase transition occurs when  $\beta = \beta_s$ . In the high temperature phase, the black hole dominates and dual field theory is in the deconfinement phase, while in the low temperature phase, the AdS soliton dominates and dual field theory is in the confined phase. The remarkable feature here is that the deconfinement transition is completely determined by the ratio  $\beta/\beta_s$ , the same as the case without the term  $\mathcal{O}(\alpha'^3)$ , although the action gets corrected by the term  $\mathcal{O}(\alpha'^3)$ .

We find that the transition point keeps unchanged in the Witten's QCD model can be understood from the point of view of the Euclidean manifold  $S^1 \times S^1 \times R^2$ , where the dual field theory resides, where one of  $S^1$  denotes the Euclidean time  $\tau$  with period  $\beta$  (inverse temperature of the black hole); the other stands for the compactified coordinate  $x_1$  with period  $\beta_s$ . The Hawking-Page transition happens when the ratio  $\beta/\beta_s$  decreases to one. When the  $\gamma W$  term is taken into account, we see from (2.20) and (4.2) that up to  $\mathcal{O}(\gamma^2)$ ,  $\beta$  and  $\beta_s$  depend on  $\gamma$  in a same manner. As a result,  $\beta/\beta_s$  is independent of  $\gamma$ ; and therefore the deconfinement transition is the same as the case without the correction term. But, clearly both of  $\beta$  and  $\beta_s$  depend on the correction term.

## 5 Conclusions and Discussions

In this paper we have discussed Hawking-Page phase transitions associated with Ricci flat  $AdS_5$  black holes with correction  $\alpha'^3 R^4$  in two situations. One is the so-called hard-wall AdS/QCD model, where a small radius region of the AdS space is removed by hand. Recently Herzog [9] has shown that deconfinement phase transition can be realized in the hard-wall QCD model via a first-order Hawking-Page transition between a Ricci flat  $AdS_5$  black hole and the AdS space in the Poincare coordinates. Removing the small radius region amounts to introducing an IR cutoff in the dual field theory. Herzog has found a relation between the deconfinement transition temperature of the holographic QCD model and the IR cutoff. The  $\alpha'$  expansion corrections in the gravity side are dual to the  $\lambda$  expansion corrections in the dual field theory with  $\lambda$  being the 't Hooft coupling constant. Therefore the leading correction  $\alpha'^3 R^4$  in the type IIB supergravity gives the  $\lambda^{-3/2}$  correction in the field theory side. We have generalized the Herzog's discussion by studying the  $\alpha'^3 R^4$  correction to the Ricci flat  $AdS_5$  black holes and found that the deconfinement temperature decreases after considering the correction (see (3.13)).

The other situation is that one of horizon coordinates is compactified for the Ricci flat  $AdS_5$  black hole. In fact this situation is just the one for Witten's QCD model

in  $2 + 1$  dimensions. In this case, a suitable reference background is the  $AdS_5$  soliton solution, which has less energy than the pure  $AdS_5$  space. We have found that the deconfinement transition of the QCD model is always determined by the ratio of inverse Hawking temperature of the black hole to the period of the compactified direction (see (4.8)). The correction  $\alpha'^3 R^4$  has no effect on the ratio  $\beta/\beta_s$ . It would be very interesting to see whether this conclusion is universal by studying the deconfinement transition for Witten's QCD model in  $3 + 1$  dimensions. Very recently, the  $\alpha'^3 R^4$  correction to the  $D4$ -brane metric has been worked out by Basu [32].

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